Microwave Measurement of Initial Properties of Ferrites using Mode Splitting Phenomenon by the Rod Resonator Method

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An accurate method of calculating the permittivity and permeability, simultaneously, of ferrites in demagnetized state is proposed in this paper. We introduce a cylindrical ferrite resonator that is located between parallel metal plates. We theoretically derive exact eigenvalue equations for the mode-splitting phenomenon of the HE₁₁₁ mode. The complex permeability and permittivity are computed from the measured resonant frequencies and quality factors of the resonator. We present an analysis of cylindrical ferrite resonator and experimental results.

Index Terms—ferrites, permeability, permittivity, demagnetization, newton method.

I. INTRODUCTION

RECENTLY, a variety of microwave applications have been developed by using ferrites. Therefore, it is very important to measure the permeability and permittivity of ferrites.

Many researches on finding the permeability and the permittivity have been done so far by the researchers. Krupka[1] introduced a measurement method for the permeability of ferrite in a demagnetized state by using resonance model. The dimension of the ferrite plays an important role regarding the accurate measurement in this model. Courtney[2] introduced a measurement method for the properties of dielectric and magnetic materials. This method is widely used to calculate the initial permeability and the permittivity of ferrites by using the TE₀₁₁ mode. However, these studies can’t measure the permeability and permittivity, simultaneously, using just one resonator.

In this paper, we introduce a simple and accurate measurement method to measure the complex permittivity and permeability at the same time, using mode splitting phenomenon of ferrites in low-loss frequency region.

II. THEORY ANALYSIS

The geometry of the resonator under consideration is shown in Fig. 1. A cylindrical ferrite resonator with a radius of \( r_0 \) and a height of \( L \) is located between two parallel metal plates. This structure has been popular for a theoretical analysis as its simple geometry requires no magnetic wall assumption and as such it can provide more accurate solutions. The formulas for the electromagnetic fields in a ferrite rod and in air can be found in [3] and will not be reproduced here. In the boundaries between the cylindrical interfaces, the well-known continuity conditions between the tangential electric and magnetic field components must be satisfied, and the conductive ground planes tangential electrical fields must vanish. By substituting expression for the electromagnetic field into boundary conditions, we obtain the system of homogeneous linear equations. The system of equations has nontrivial solutions only if the corresponding determinant of the system vanishes.

The initial permeability and permittivity of ferrites can be measured simultaneously using mode splitting phenomenon because two characteristic equations about \( HE_{+111} \) and \( HE_{-111} \) exist in a magnetized state. Although the ferrite has permeability tensor at magnetized state, we can assume that the permeability of ferrites of very small magnetization is approximately equal to the permeability at demagnetized state[4]. The initial permeability \( \bar{\mu} ( = \mu' - j\mu'' ) \) and permeability \( \bar{\varepsilon} ( = \varepsilon' - j\varepsilon'' ) \) are found to be solutions to the following system of nonlinear equations:

\[
\begin{align*}
F(\varepsilon', \mu', f_{HE_{+111}}) &= 0 \\
F(\varepsilon', \mu', f_{HE_{-111}}) &= 0
\end{align*}
\]

where, \( F \) is the resonance mode eigenvalue equation in determinant form for these resonators, \( f_{HE_{+111}} \) and \( f_{HE_{-111}} \) are the measured resonant frequencies of the \( HE_{+111} \) and \( HE_{-111} \) modes respectively for the resonators containing magnetized ferrite sample at a certain static external magnetic field intensity. To calculate two unknown coefficients, the well-known newton iteration method that is a numerical procedure has been utilized. As a result, the initial permeability and the permittivity can be calculated at demagnetized state.

The imaginary parts of the initial permeability tensor components can be found after computing their real parts as the solutions to the following system of linear equations:

\[
\begin{align*}
(Q^*)^{-1} &= (Q_{\varepsilon})^{-1} + p_n^*(\varepsilon'' / \varepsilon') + p_{\mu}^*(\mu'' / \mu') \\
(Q^*)^{-1} &= (Q_{\mu})^{-1} + p_n^*(\varepsilon'' / \varepsilon') + p_{\mu}^*(\mu'' / \mu')
\end{align*}
\]

where \( Q^* \) and \( Q^- \) are the unloaded Q factors for the \( HE_{+111} \), and \( HE_{-111} \) mode, respectively. \( Q_{\varepsilon}^* \) and \( Q_{\mu}^- \) are the Q factors depending on conductor losses in metal plates for the \( HE_{+111} \).
and $HE_{-111}$ mode, respectively. $p_x^+$ and $p_x^-$ are the electric or magnetic energy filling factors computed by the perturbation method as follows:

$$p_x^i = 2\left|\delta f_{HE_{+111}} / \delta x\right| / f_{HE_{+111}} \tag{3}$$

Where $x$ denotes $\varepsilon'$ and $\mu'$, respectively. The differentials appeared in the above expressions were computed numerically on the basis of the corresponding eigenvalue equations.

### III. Measurement and Circulation

We used disc-shaped samples that are composed of Li-ferrite (3-2002 from Pacific ceramics). Samples have the same radius $r_0 = 3.83\text{mm}$ and height $L = 6.5\text{mm}$, $7.5\text{mm}$, $8.5\text{mm}$ and $9.5\text{mm}$, respectively. The saturation magnetization of Li-ferrite is 1960 Gauss. The measured reflection coefficient results obtained at X-band are shown in Fig. 2. In order to observe the mode splitting behavior, the static magnetic field was applied to ferrite resonator using a Helmholtz coil that offers homogenous dc magnetic field. Initially, we observed the $HE_{+111}$ mode (the first mode) in the demagnetized state. After applying a very small static magnetic bias (1 mT), the lowest $HE_{-111}$ mode and $HE_{+111}$ mode are shown in the following lower frequency order.

The complex relative permittivity and permeability in the completely demagnetized state were measured using the cavity perturbation method (CPM) to verify the results of the proposed method. The Fig. 3 and Fig. 4 show the calculated permittivity and permeability using the proposed method and the measured result by CPM. When the continuity of permittivity is considered, the permittivity results of proposed method using $HE_{+111}$ mode correspond to the perturbation result as shown in Fig. 3. In addition, we compared with the measured permeability by CPM with the calculated permeability by scholmann’s theory [5] as shown in Fig. 4. The calculated values of the permeability were closely corresponded to the measured results. In addition, the imaginary part of properties is also very reasonable results, considering that the low-loss ferrite was used. Consequently, we can derive a conclusion that the proposed method is reliable for measuring the complex permittivity and permeability of ferrites in the demagnetized state.

### IV. Conclusion

In this paper, we have proposed a simple method for measuring permeability and permittivity of ferrites, simultaneously, at demagnetized state using mode splitting phenomenon. To determine the initial properties of ferrites, the cylindrical ferrite resonator between two parallel metal plates has been analyzed. Using a Li-ferrite resonator we have measured resonant frequencies and calculated the value of initial properties, at the same time. The calculated results have been compared with the measured result of CPM. We have verified that the calculated properties closely match the experiment results.

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### REFERENCES


